(July 1996)

A Model-independent Way to Measure $|V_{ub}/V_{cb}|$

C. S. Kim*
Department of Physics, Yonsei University Seoul 120-749, Korea†
and
Theory Division, KEK, Tsukuba, Ibaraki 305, Japan‡

We propose a new model-independent method, to determine the ratio $|V_{ub}/V_{cb}|$, which is theoretically described by the phase space factor and the well-known perturbative QCD correction only. We explore the possible experimental options: the measurement of inclusive hadronic invariant mass distributions, the '$D-\pi$' separation condition. We also discuss the relevant experimental backgrounds.

1. General Discussions The CKM matrix element V_{ub} is important to the SM description of CP-violation. If it were zero, there would be no CP-violation from the CKM matrix elements (*i.e.* in the SM), and we have to seek for other source of CP violation in $K_L \to \pi\pi$. Observations of semileptonic $b \to u$ transitions by the CLEO [1] and ARGUS [2] imply that V_{ub} is indeed nonzero, and it is important to extract the modulus $|V_{ub}|$ from semileptonic decays of B mesons as accurately as possible.

Presently, the charged lepton energy spectrum ($d\Gamma/dE_l$) has been measured, and the $b \to u$ events are selected from the high end of the charged lepton energy spectrum. This method is applied to both inclusive and exclusive semileptonic B decays. However, this cut on E_l is not very effective, since only below 10% of $b \to u$ events survive this cut at the B meson rest frame. (In the future asymmetric B-factories with boosted B mesons,

*Talk given at Asian Science Seminar (KOSEF-JSPS Winter School), Seoul Korea, on February 21 - 28 (1996). Proceedings will be published by J. Kor. Phys. Soc. (1996).
†kim@cskim.yonsei.ac.kr
‡cskim@kekvax.kek.jp

even much less than 10% would survive the E_l cut over the $b \to c$ threshold.) We also note that the dependences of the lepton energy spectrum on perturbative and non-perturbative QCD corrections [3,4] as well as on the unavoidable specific model parameters (*e.g.* the parameter $p_{_F}$ of ACCMM model [5]) are the strongest at the end point region, which makes the model-independent determination of $|V_{ub}/V_{cb}|$ almost impossible from the inclusive distribution of $d\Gamma/dE_l$. For exclusive $B \to X_u l\nu$ decays, the application of heavy quark effective theory (HQET) is much limited, since u-quark is not heavy compared to Λ_{QCD}. And the theoretical predictions for the required hadronic matrix elements are largely different depending on which model we use, as can be seen in the following, as an example, for $\bar{B}^0 \to \rho^+ l^- \bar{\nu}$,

$$
\begin{aligned}
\gamma_\rho \equiv \frac{\Gamma_{theory}(\bar{B}^0 \to \rho^+ l^- \bar{\nu})}{|V_{ub}|^2} &= \quad 8.3 \times 10^{12}/sec \ , \quad (\ [6]) \\
&= 32.9 \times 10^{12}/sec \ , \quad (\ [7]) \\
&= 18.7 \times 10^{12}/sec \ . \quad (\ [8])
\end{aligned}
$$

Recently it has been suggested that the measurements of hadronic invariant mass spectrum [9] as well as hadronic energy spectrum [10] in the inclusive $B \to X_{c(u)} l\nu$ decays can be useful in extracting $|V_{ub}|$ with better theoretical understandings. In future asymmetric B-factories with microvertex detector, the hadronic invariant mass spectrum will offer alternative ways to select $b \to u$ transitions that are much more efficient than selecting the upper end of the lepton energy spectrum, with much less theoretical uncertainties. The measurement of ratio $|V_{ub}/V_{ts}|$ from the differential decay widths of the processes $B \to \rho l\nu$ and $B \to K^* l\bar{l}$ by using $SU(3)$-flavor

symmetry and the heavy quark symmetry has been also proposed [11]. It is urgently important that all the available methods of determinating V_{ub} have to be thoroughly explored to measure the most important CKM matrix element as accurately as possible in the forthcoming B-factories.

2. Theoretical Discussions Over the past few years, a great progress has been achieved in our understanding of *inclusive* semileptonic decays of heavy mesons [4], especially in the lepton energy spectrum. However, it turns out that the end point region of the lepton energy spectrum cannot be described by $1/m_Q$ expansion. Rather, a partial resummation of $1/m_Q$ expansion is required [12], closely analogous to the leading twist contribution in deep inelastic scattering, which brings about possible model dependences.

Even with a theoretical breakdown near around the end point region of lepton energy spectrum, accurate predictions of the **total** integrated semileptonic decay rate have been obtained [4] including the first non-trivial non-perturbative corrections as well as radiative perturbative QCD correction [3]. The related uncertainties in calculation of the integrated decay rate have been also analyzed [13–15]. The total inclusive semileptonic decay rate for $B \to X_q l\nu$ is given [14] as

$$\begin{aligned}
&\Gamma\left(B \to X_q l\nu\right) = \frac{G_F^2 m_b^5}{192\pi^3}|V_{qb}|^2 \\
&\Bigg\{ \left[z_0(x_q \equiv m_q/m_b) - \frac{2\alpha_s(m_b^2)}{3\pi} g(x_q) \right] \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} \right) \\
&- z_1(x_q)\frac{\mu_G^2}{m_b^2} + \mathcal{O}(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3) \Bigg\} , \qquad (0.1)
\end{aligned}$$

where

$$z_0(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \log x \quad ,$$

$$z_1(x) = (1 - x^2)^4 \quad ,$$

$$\text{and} \quad g(x) = (\pi^2 - 31/4)(1 - x)^2 + 3/2$$

is the corresponding single gluon exchange perturbative QCD correction [3,16]. The expectation value of energy due to the chromomagnetic hyperfine interaction, μ_G, can be related to the $B^* - B$ mass difference

$$\mu_G^2 = \frac{3}{4}(M_{B^*}^2 - M_B^2) \approx (0.350 \pm 0.005) \text{ GeV}^2 \quad , \qquad (0.2)$$

and the expectation value of kinetic energy of b-quark inside B meson, μ_π^2, is given from the various arguments [17–19],

$$0.30 \text{ GeV}^2 \leq \mu_\pi^2 \leq 0.65 \text{ GeV}^2 \quad , \qquad (0.3)$$

which shows much larger uncertainties compared to μ_G^2. The value of $|V_{cb}|$ has been estimated [13–15] from Eq. (0.1) of the total decay rate $\Gamma(B \to X_c l \nu)$ by using the pole mass of m_b and a mass difference $(m_b - m_c)$ based on the HQET. As can be easily seen from Eq. (0.1), the factor m_b^5, which appears in the semileptonic decay rate, but not in the branching fraction, is the largest source of the uncertainty, resulting in about $5 \sim 20\%$ error in the prediction of $|V_{cb}|$ via the semileptonic branching fraction and B meson life time. Historically, the ACCMM model [5] was motivated to avoid this factor m_b^5, and at the same time to incorporate the bound state effect of initial B meson.

The ratio of CKM matrix elements $|V_{ub}/V_{cb}|$ can be determined in a model-independent way by taking the ratio of semileptonic decay widths $\Gamma(B \to X_u l \nu)/\Gamma(B \to$

$X_c l\nu$). As can be seen from Eq. (0.1), this ratio is theoretically described by the phase space factor and the well-known perturbative QCD correction only,

$$\frac{\Gamma(B \to X_u l\nu)}{\Gamma(B \to X_c l\nu)} = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \left[1 - \frac{2\alpha_s}{3\pi}\left(\pi^2 - \frac{25}{4}\right)\right] \times \left[z_0(x_c) - \frac{2\alpha_s}{3\pi} g(x_c)\right]^{-1} . \quad (0.4)$$

We strongly emphasize here that the sources of the main theoretical uncertainties, the most unruly factor m_b^5 and the still-problematic non-perturbative contributions, are all canceled out in this ratio. By taking $\alpha_s(m_b^2) = (0.24 \pm 0.02)$, and using the mass difference relation from the HQET [20], which gives $x_c \equiv m_c/m_b \approx 0.25 - 0.30$, the ratio of the semileptinic decay widths is estimated as

$$\frac{\Gamma(B \to X_u l\nu)}{\Gamma(B \to X_c l\nu)} \simeq (1.83 \pm 0.28) \times \left|\frac{V_{ub}}{V_{cb}}\right|^2 , \quad (0.5)$$

and the ratio of CKM elements is

$$\left|\frac{V_{ub}}{V_{cb}}\right| \simeq (0.74 \pm 0.06) \times \left[\frac{\mathcal{B}(B \to X_u l\nu)}{\mathcal{B}(B \to X_c l\nu)}\right]^{1/2} , \\ \simeq (0.75 \pm 0.06) \times \left[\frac{\mathcal{B}(B \to X_u l\nu)}{\mathcal{B}(B \to X l\nu)}\right]^{1/2} , \quad (0.6)$$

where in the last relation we have assumed $\mathcal{B}(B \to X l\nu) \sim (1.02) \cdot \mathcal{B}(B \to X_c l\nu)$. Once the ratio of semileptonic decay widths (or equivalently the ratio of branching fractions $\mathcal{B}(B \to X_u l\nu)/\mathcal{B}(B \to X_c l\nu)$) is measured in the forthcoming asymmetric B-factories, this will give a powerful model-independent determination of $|V_{ub}/V_{cb}|$. We will discuss on this experimental possibility in details in the next Section. There is absolutely no model dependence in these ratios Eqs. (0.4,0.5,0.6). As explained earlier, for example in ACCMM model [5] the model dependence comes in via the introduction of parameter p_F

to avoid the factor m_b^5, which is now canceled in these ratios. The problem of the semileptonic branching fraction (*so-called* the discrepancy between the theoretical prediction and the actual experimental measurement of the semileptonic branching fraction $\mathcal{B}_{sl}$) would be also canceled out in the ratio of the branching fractions.

3. Experimental Discussions In order to measure $|V_{ub}/V_{cb}|$ (and $|V_{ub}|$) model-independently by using the relations Eq. (0.6), it is critically required to separate the $b \to u$ semileptonic decays from the dominant $b \to c$ semileptonic decays, and to precisely measure branching fraction $\mathcal{B}(B \to X_u l\nu)$ or the ratio $\mathcal{B}(B \to X_u l\nu)/\mathcal{B}(B \to X_c l\nu)$. At presently existing symmetric B-factories, ARGUS and CLEO, where B and $\bar{B}$ are produced almost at rest, this required separation is possible only in the very end point region of lepton energy spectrum, because both B and $\bar{B}$ decay into whole 4π solid angle from the almost same decay point, and it is not possible for the produced particles to be identified from which is the original B meson. However, in the forthcoming asymmetric B-factories with microvertex detectors, BABAR and BELLE [21], where the two beams have different energies and the produced $\Upsilon(4S)$ is not at rest in the laboratory frame, the bottom decay vertices too will be identifiable with still greater advantage to the analyses. The efficiency for the whole event reconstruction could be relatively high (maybe $1 \sim 10\%$ efficiency) limited only by about 60% of π^0-reconstruction efficiency, and this $b \to u$ separation would be experimentally viable option.

As of the most straightforward separation method, the

measurements of inclusive hadronic invariant mass (m_X) distributions in $B \to X_{c,u} l\nu$ can be very useful for the fully reconstructed semileptonic decay events. For $b \to c$ decays, one necessarily has $m_X \geq m_D = 1.86$ GeV. Therefore, if we impose a condition $m_X < m_D$, the resulting events come only from $b \to u$ decays, and about 90% of the $b \to u$ events survive this cut. This is already in sharp contrast with the usual cut on charged lepton energy E_l. In fact, one can relax the condition $m_X < m_D$, and extract the total $b \to u$ semileptonic decay rate [9], because the m_X distribution in $b \to c$ decays is completely dominated by contributions of three resonances D, D^* and D^{**}, which are essentially like δ-functions,

$$\frac{d\Gamma}{dm_X} = \Gamma(B \to Rl\nu)\ \delta(m_X - m_R) \quad , \tag{0.7}$$

where the resonance $R = D, D^*$ or D^{**}. In other words, one is allowed to use the $b \to u$ events in the region even above $m_X \geq m_D$, first by excluding small regions in m_X around $m_X = m_D, m_{D^*}, m_{D^{**}}$, and then by including the regions again numerically in the m_X distribution of $b \to u$ decay from its value just around the resonances. We note that there is possibly a question of bias. Some classes of final states (*e.g.* those with low multiplicity, few neutrals) may be more susceptible to a full and unambiguous reconstruction. Hence an analysis that requires this reconstruction may be biassed. However, the use of topological information from microvertex detectors should tend to reduce the bias, since vertex resolvability depends largely on the proper time of the decay and its orientation relative to the initial momentum (that are independent of the decay mode). Also such a bias can be

allowed for in the analyses, via a suitable Monte Carlo modeling. For more details on this inclusive hadronic invariant mass distribution $d\Gamma/dm_X$, please see Ref. [9].

Even without full reconstructions of final particles, one can separate $b \to u$ decays from $b \to c$ decays by using the particle decay properties [22]. Since $D^{**} \to D^* + \pi$ and $D^* \to D + \pi$, the semileptonic $b \to c$ decays always produce at least one final state D meson, compared to $b \to u$ decays which produce particles, π, ρ, ... that always decay to one or more π mesons at the end. Therefore, the $b \to u$ decay separation can be achieved only with the accurate '$D-\pi$' separation in particle detectors. There still is a possible non-resonant decay background from $B \to (D+\pi)l\nu$ in using previously explained inclusive m_X distribution separation. However, with this addtional '$D-\pi$' separation condition the $b \to u$ decays can be safely differentiated from the dominant $b \to c$ decays. There is another possible source of background to this '$D-\pi$' separation condition from the cascade decay of $b \to c \to sl\nu$. Recently ARGUS and CLEO [23] have separated this cascade decay background from the signal events to extract the model-independent spectrum of $d\Gamma/dE_l(B \to X_c l\nu)$ for the whole region of electron energy, by taking care of lepton charge and $B-\bar{B}$ mixing systematically. In future asymmetric B-factories with much higher statistics, this cascade decay will not be any serious background at all except for the case with very low energy electron.

In view of the potential importance of $\mathcal{B}(B \to X_u l\nu)/\mathcal{B}(B \to X_c l\nu)$ as a new theoretically model-

independent probe for measuring $|V_{ub}|$ and $|V_{ub}/V_{cb}|$, we would like to urge our experimental colleagues to make sure that this $b \to u$ separation can indeed be observed.

Acknowledgements

The work was supported in part by the Korean Science and Engineering Foundation, Project No. 951-0207-008-2, in part by Non-Directed-Research-Fund, Korea Research Foundation 1993, in part by Yonsei University Faculty Research Grant, and in part by the Basic Science Research Institute Program, Ministry of Education 1995, Project No. BSRI-95-2425. For more details on this talk, please see Ref. [24].

---

[1] CLEO Collab.: R. Fulton *et al.*, Phys. Rev. Lett. **64**, 16 (1990); J. Bartelt *et al.*, Phys. Rev. Lett. **71**, 4111 (1993).
[2] ARGUS Collab.: H. Albrecht *et al.*, Phys. Lett. **B234**, 409 (1990); *ibid.* **B241**, 278 (1990); *ibid.* **B255**, 297 (1991).
[3] N. Cabibbo, G. Corbò and L. Maiani, Nucl. Phys. **B155**, 93 (1979); G. Corbò, *ibid.* **B212**, 99 (1983); M. Jezabek and J.H. Kühn, *ibid.* **B320**, 20 (1989); A. Czarnecki and M. Jezabek, *ibid.* **B427**, 3 (1994).
[4] I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993); A.V. Manohar and M.B. Wise, Phys. Rev. **D49**, 1310 (1994); B. Blok, L. Koyrakh, M. Shifman and A.I. Vainshtein, Phys. Rev. **D49**, 3356 (1994); A. Falk, M. Luke and M. Savage, Phys. Rev. **D49**, 3367 (1994); T. Mannel, Nucl. Phys. **B423**, 396 (1994); M. Neubert, Phys. Rev. **D49**, 3392 (1994).
[5] G. Altarelli, N. Cabibbo, G. Corbò, L. Maiani and G. Martinelli, Nucl. Phys. **B208**, 365 (1982).
[6] N. Isgur, D. Scora, B. Grinstein and M.B. Wise, Phys. Rev. **D39**, 799 (1989); D. Scora and N. Isgur, Phys. Rev. **D52**, 2783 (1995).
[7] J.G. Körner and G.A. Schuler, Z. Phys. **C38**, 511 (1988); *ibid.* **C46**, 93 (1990).
[8] M. Wirbel, B. Stech and M. Bauer, Z. Phys. **C29**,
[9] V. Barger, C.S. Kim and R.J.N. Phillips, Phys. Lett. **B235**, 187 (1990); *ibid.* **B251**, 629 (1990); C.S. Kim, D.S. Hwang, P. Ko and W. Namgung, Nucl. Phys. B

(Proc. Suppl.) **37A**, 69 (1994); Phys. Rev. **D50**, 5762 (1994).

[10] A.O. Bouzas and D. Zappalà, Phys. Lett. **B333**, 215 (1994).

[11] A.I. Sanda and A. Yamada, Phys. Rev. Lett. **75**, 2807 (1995).

[12] I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Int. J. Mod. Phys. **A9**, 2467 (1994); T. Mannel and M. Neubert, Phys. Rev. **D50**, 2037 (1994).

[13] M. Luke and M.J. Savage, Phys. Lett. **B321**, 88 (1994).

[14] M. Shifman, N.G. Uraltsev and A. Vainshtein, Phys. Rev. **D51**, 2217 (1995); (Erratum) Phys. Rev. **D52**, 3149 (1995).

[15] P. Ball, M. Beneke and V.M. Braun, Phys. Rev. **D52**, 3929 (1995).

[16] C.S. Kim and A.D. Martin, Phys. Lett. **B225**, 186 (1989).

[17] P. Ball and V.M. Braun, Phys. Rev. D **49**, 2472 (1994); I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. D **52**, 196 (1995); E. Bagan, P. Ball, V.M. Braun and P. Gosdzinsky, Phys. Lett. **B342**, 362 (1995).

[18] D.S. Hwang, C.S. Kim and W. Namgung, Z. Phys. **C69**, 107 (1995); hep-ph/9506476, to be published in Phys. Rev. **D** (1996); hep-ph/9604225 (1996).

[19] A. Kapustin, Z. Ligeti and M.B. Wise, B. Grinstein, hep-ph/9602262, to be published in Phys. Lett. **B** (1996); M. Gremm , A. Kapustin, Z. Ligeti and M.B. Wise, hep-ph/9603314 (1996).

[20] E. Eichten and B. Hill, Phys. Lett. **B234**, 511 (1990); H. Georgi, Phys. Lett. **B240**, 447 (1990); C. Quigg and J.L. Rosner, Phys. Rev. **D23**, 2625 (1981).

[21] BABAR Technical Design Report, SLAC-R-95-457 (March 1995); BELLE Technical Design Report, KEK Report 95-1 (April 1995).

[22] Particle Data Group: K. Hikasa *et al.*, Phys. Rev. **D50**, 1173 (1994).

[23] CLEO Collab.: B. Barish *et al.*, Phys. Rev. Lett. **76**, 1570 (1996); ARGUS Collab.: H. Albrecht *et al.*, Phys. Lett. **B318**, 397 (1993)

[24] C.S. Kim, hep-ph/9605201, KEK Preprint 96-6 (May 1996).